Suppression of spiral waves and spatiotemporal chaos by generating target waves in excitable media

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(Received 7 June 2002; revised manuscript received 12 March 2003; published 29 August 2003)

A method for suppressing spiral waves and spatiotemporal chaos in excitable media is proposed. Applying suitable periodic force to a single point, we can successfully suppress spiral waves as well as spatiotemporal chaos by generating target waves. After we turn off the external force, target waves finally disappear and the whole system which was in the state of spiral wave or spatiotemporal chaos goes to the spatially homogeneous steady state. It is shown that our control method is not sensitively model dependent. It works for a model for catalytic CO oxidation on platinum as well as for a model for cardiac muscle.

DOI: 10.1103/PhysRevE.68.026134 PACS number(s): 82.40.Ck, 05.45.Gg, 47.27.Rc, 47.54.+r

Spiral waves and spatiotemporal chaos (defect-mediated turbulence) exist popularly in excitable media, such as cardiac muscle, the oxidation of CO on platinum, and reacting chemical systems, for example, the Belousov-Zhabotinsky (BZ) reaction [1]. In some cases spiral waves and spatiotemporal chaos are undesirable because of their harmfulness. For example, spirals in cardiac muscle play an essential role in heart diseases such as arrhythmia—ventricular fibrillation, the major reason behind sudden cardiac death, is turbulent cardiac electrical activity in which rapid, irregular disturbances in the spatiotemporal electrical activation of the heart make it incapable of any concerted pumping action. Therefore, suppression of spiral waves and spatiotemporal chaos in excitable media is of much practical interest [2–12]. Aranson, Levine, and Tsimring [9] suggested a method of spatiotemporal turbulence control in a model for catalytic CO oxidation on Pt(110) by developing a spiral wave with local feedback injection. In Ref. [10], the authors used time-delayed feedback signals injected into uniformly distributed local point, to prevent spiral wave from breaking up to spatiotemporal turbulence. Using low pulses over a coarse mesh of lines, Sinha, Pande, and Pandit [11] achieved control of spatiotemporal turbulence (ventricular fibrillation) in a model for cardiac muscle. Experimentally, Kim *et al.* [12] successfully suppressed chemical turbulence in catalytic CO oxidation on Pt(110) by global delayed feedback. In this paper, we will consider the suppression of spiral waves and spatiotemporal turbulence in excitable media via periodic forcing (open loop control) at a single point by developing target waves.

Let us begin with an activator controller two-variable reaction-diffusion model for catalytic CO oxidation on Pt(110) [13]:

$$\partial u/\partial t = -\frac{1}{\epsilon}u(u-1)[u-(v+b)/a]+\nabla^2 u, \tag{1a}$$

$$\partial v/\partial t = f(u)-v, \tag{1b}$$

where $f(u)=0$ if $0\leqslant u<1/3$; $f(u)=1-6.75u(u-1)^2$ if $1/3\leqslant u\leqslant 1$; and $f(u)=1$ if $u>1$. Here u and v describe the activator and the inhibitor variables, respectively. The spatiotemporal dynamics [we consider two-dimensional case, $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$] is investigated by varying ϵ and fixing $a=0.84$ and $b=0.07$. In the range $0.01<\epsilon<0.06$, suitable initial conditions lead to steadily rotating spiral waves. At $\epsilon=0.06$, the spiral waves undergo a transition from steady rotation to meandering. In the range $\epsilon>0.07$, spiral waves will break up and the system will quickly fall into a turbulence state.

We first take $\epsilon=0.04$, where system (1) can have a steadily rotating spiral wave [Fig. 1(a)]. Our task is to suppress this undesirable spiral pattern. To control system (1), one can add an external field to Eqs. (1) [4,6,9,11,14]. Here our strategy is to apply a periodic signal to a small fixed area in the system for the control purpose, i.e., add to the right-hand side of Eq. (1a) the term $\Gamma\delta_{i,\mu}\delta_{j,\nu}\cos(i\omega t)$, where i,j are the integer numbers corresponding to the discretized x and y variables as $x_i=(i-1)\Delta x$, $y_i=(j-1)\Delta y$, respectively; μ,ν are integer numbers. The control area is taken as

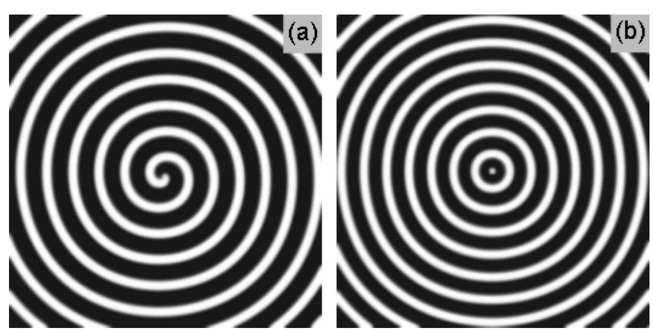

FIG. 1. Spiral wave suppression by developing target waves with a periodic signal injection of model (1). $\epsilon=0.04$, $n=5$, $\Gamma=1.0$, $\omega=1.807$. (a) $t=0$, (b) $t=400$ time units. The system size is 100×100, Grid 256×256 points, and $\Delta t=0.02$. No-flux boundary conditions are imposed.

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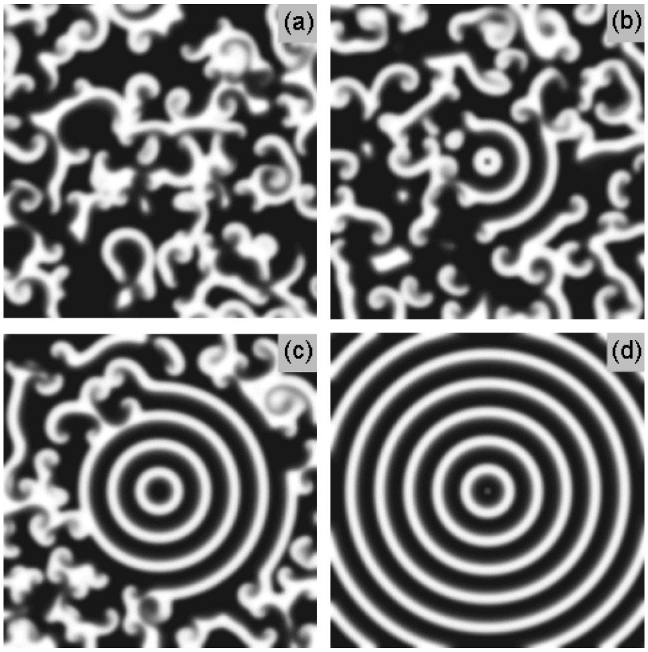

FIG. 2. Suppression of spatiotemporal chaos by local periodic injection of system (1). $\epsilon=0.085$, $n=5$, $\Gamma=2.0$, $\omega=1.205$. (a) $t=0$, (b) $t=200$ time units, (c) $t=400$ time units, and (d) $t=1000$ time units.

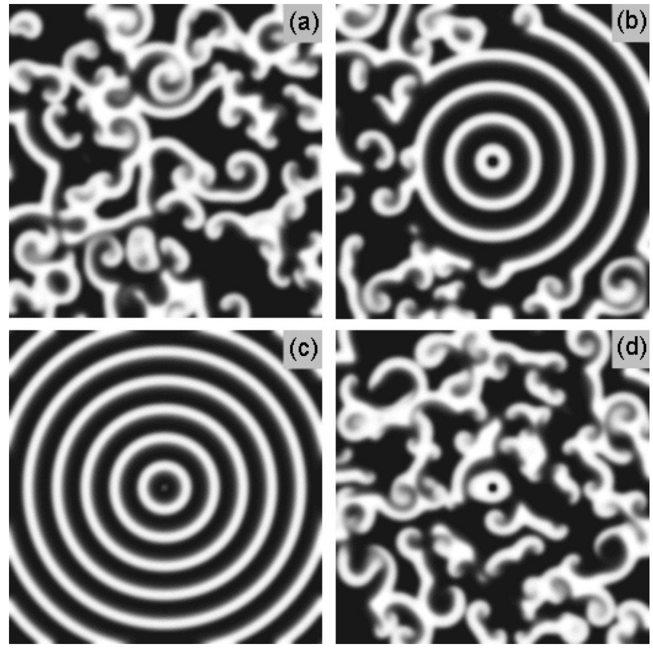

FIG. 3. The states ($t=1000$ t.u.) of Eqs. (1) with $\epsilon=0.085$, $n=5$, $\Gamma=2.0$ for different control angular frequencies: (a) $\omega=1.084$, (b) $\omega=1.114$, (c) $\omega=1.205$, and (d) $\omega=1.265$.

a square in the space center with $n\times n$ sites (for $n=1$, $\mu,\nu=128$; for $n=2$, $\mu,\nu=127,128$ etc.). Generally, an external injection into a small local region cannot essentially change the pattern of Fig. 1(a), the spiral remains the same with slight deformation only. We have tested the local control effects for wide ranges of Γ and ω, and find that under certain conditions, the system state can be dramatically changed by the local periodic forcing, specifically, target waves can be generated in replacing the original spiral wave. In Fig. 1, we inject a local periodic signal into a perfect spiral. The spiral tip is gradually driven out of the boundary, and the system is finally dominated by target waves. In some other realistic cases, it is important to suppress spiral waves and realize stationary homogeneous state in excitable media [2–5]. With our control approach we can readily do so by releasing the control after target wave state is realized. After we turn off the external force from Fig. 1(b), no new target waves can be generated afterwards and the former target waves will move out of the boundary. Finally, the whole system evolves to the spatially homogeneous steady state $[u(x,y,t)=0,v(x,y,t)=0]$.

We now turn our attention to spatiotemporal chaos, or say defect-mediated turbulence [$\epsilon=0.085$, Fig. 2(a)], and apply the local periodic forcing method for the turbulence control. For certain suitable Γ and ω, we can again generate target waves and develop them to annihilate spatiotemporal chaos in the whole system. Taking $t=0$ [see Fig. 2(a)] as the initial condition and injecting a periodic signal with $n=5$, $\Gamma=2.0$, and $\omega=1.205$ into the central point of the system, we find that a small target wave moving outward is generated near the central part [see Fig. 2(b)], and target waves generated continuously from the forced point can ceaselessly drive the defects out of the system [see Fig. 2(c)]. Finally, in Fig. 2(d) the whole space is firmly controlled by the target waves while the system parameters remain in spatiotemporal chaos regime. The control efficiency in Fig. 2 is surprisingly high and the approach is rather simple. We use only one single signal, which is not very large (the forcing amplitude Γ is of the same order as the u, v variable values), injecting into 5×5 space sites area (extremely small in comparison with the whole turbulent region of 256×256 sites) which turns the violent turbulence to a perfect regular target wave.

It is then interesting to understand the mechanism underlying the above high control efficiency, and the facts affecting the control results. First, spatiotemporal chaos can be controlled successfully by local control in an $n\times n$ space area with n varying in a wide range. In particular, we can control the defect-mediated turbulence, as in Fig. 2(a), by injecting only one site among the 256×256 sites. Second, for successful control of turbulence, the amplitude of external force Γ should not be small. This can be easily accepted since too weak control signal cannot inject enough "energy" to generate target waves and to annihilate violent spatiotemporal chaos. We have numerically test different control strengths for various control areas $n\times n$. For each n, there exists a minimal amplitude $\Gamma^n_{\min}$, and successful turbulence control can be achieved only for $\Gamma>\Gamma^n_{\min}$. For example, we have $\Gamma^1_{\min}=30.1$, $\Gamma^5_{\min}=0.92$, and $\Gamma^{10}_{\min}=0.34$ for $\omega=1.205$. Third, the angular frequency ω of the signal should be properly chosen. In Fig. 3 we show the system states at $\Gamma=2.0$, $n=5$ for different ω's. The successful control can be achieved only for certain resonant frequency [Fig. 3(c)]; too small [Figs. 3(a) and 3(b)] and too large [Fig. 3(d)] frequencies do not give good control results. Our numerical results show that the mechanism is integer-times-frequency resonance. The single frequency resonance ($\omega=\Omega$ where Ω is

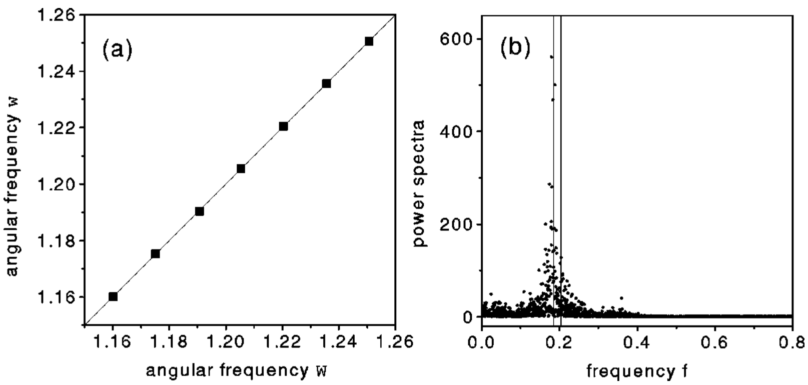

FIG. 4. (a) The resonant relation between external forcing and target waves, the solid line is the function $\omega=\Omega$. We change ω in a small range near 1.205 with $\epsilon=0.085$, $n=5$, $\Gamma=4.0$. (b) The power spectra of one site of system (1) without control, $\epsilon=0.085$. We can successfully control the spatiotemporal chaos only within the frequencies between the two vertical lines with $n=5$, $\Gamma=4.0$.

the angular frequency of generated target waves) relations between external force and target waves are clearly shown in Fig. 4. We can also obtain successful control of spatiotemporal chaos for double ($\omega=2\Omega$) and triple ($\omega=3\Omega$) frequency resonances. Now the mechanism of our control approach of suppressing spatiotemporal chaos can be well heuristically understood. By injecting periodic force into a small space area, one can generate certain target wave center via various resonant stimulations. This center plays a role of target wave source, which continually emits outward target waves. Target waves have a property to propagate the motion of source region along the radial direction. With this propagation, the target wave state itself can transmit the effect of control signal from the forcing region to far away along the radial direction, and fully suppress spatiotemporal chaos in the whole space.

To check whether our control method is sensitively model dependent, we apply our method to a model for cardiac tissue [15,11]:

$$\partial e/\partial t=\nabla^2 e-f(e)-g, \tag{2a}$$

$$\partial g/\partial t=\varepsilon(e,g)(ke-g), \tag{2b}$$

with $f(e)=C_1 e$ when $e<e_1$; $f(e)=-C_2 e+a$ when $e_1\leqslant e\leqslant e_2$; $f(e)=C_3(e-1)$ when $e>e_2$, and $\varepsilon(e,g)=\varepsilon_1$ when $e<e_2$; $\varepsilon(e,g)=\varepsilon_2$ when $e>e_2$; $\varepsilon(e,g)=\varepsilon_3$ when $e<e_1$ and $g<g_1$. Here $e_1=0.0026$, $e_2=0.837$, $C_1=20$, $C_2=3$, $C_3=15$, $a=0.06$, $k=3$, $g_1=1.8$, $\varepsilon_1=1/75$, $\varepsilon_2=1.0$, and $\varepsilon_3=0.3$. With these parameter values, system (2) is in the spatiotemporal chaos state [corresponding to fibrillation in cardiac muscle, see Fig. 5(a)]. Applying a periodic signal to a small fixed area [adding to the right-hand side of Eq. (2a) the term $\Gamma\delta_{i,\mu}\delta_{j,\nu}\cos(i\omega t)$], we find similar control results (see Fig. 5 and a successful control is obviously achieved). After we turn off the control signal [$t=4100$ time units, see Fig. 5(b), as the initial condition], target waves will not break up. All of them move towards the system boundary and finally disappear. The final state of the whole system is the spatially homogeneous steady state [$e(x,y,t)=0, g(x,y,t)=0$]. So, we also realize controlling spatiotemporal chaos to the spatially homogeneous steady state with local injection. It will be significant to realize our simple control method (local periodic forcing) in controlling ventricular fibrillation which is the leading cause of sudden cardiac death.

We also check whether our method is against noise because there are many realistic situations in which noise cannot be neglected [16]. For confirming this conclusion, we add spatiotemporal white noise $\sigma(x,y,t)[\langle\sigma(x,y,t)\rangle=0, \langle\sigma(x,y,t)\sigma(x',y',t')\rangle=D\delta(x-x')\delta(y-y')\delta(t-t')]$ to the right-hand side of Eq. (1a). Though noise is applied to all the system while periodic forcing is applied to only a very small area, turbulence is still successfully replaced by target waves with slight irregular deformation [see Fig. 6]. Numerical simulations show that our control method works well when noise amplitude $D<0.01$.

Sinha, Pande, and Pandit [11] achieved controlling spatiotemporal turbulence in cardiac muscle [the same model as used in this paper, Eqs. (2) with same parameter values] to spatially homogeneous steady state. For the system in Fig.

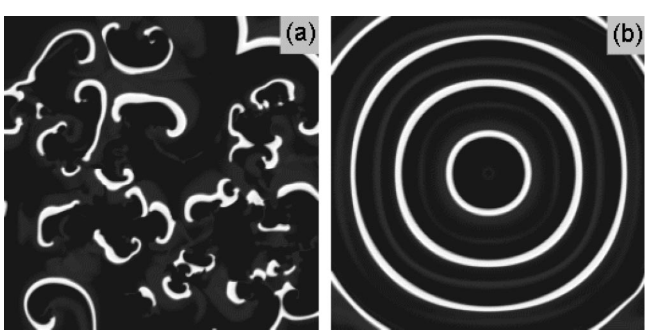

FIG. 5. Elimination of spatiotemporal chaos (fibrillation) in the model for cardiac muscle (2) with local injection, $n=6$, $\Gamma=6.0$, $\omega=0.82$. (a) $t=0$, (b) $t=4100$ time units. The system size is 250×250, grid 500×500 points, and $\Delta t=0.01$.

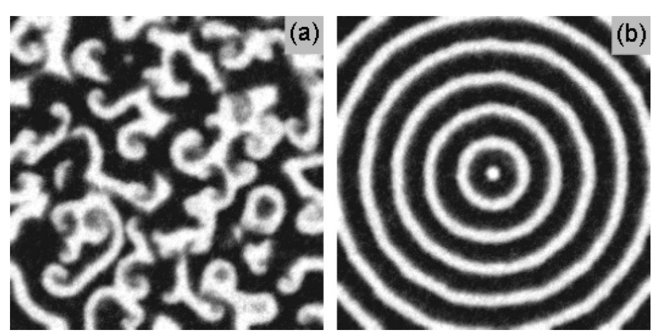

FIG. 6. Spatiotemporal chaos suppression by local periodic injection of model (1) with spatiotemporal white noise. $\epsilon=0.085$, $n=5$, $\Gamma=2.4$, $\omega=1.145$, $D=0.008$. (a) $t=0$, (b) $t=1800$ time units.

5(a), they need to apply a pulse to the e field, a mesh composed of eight lines of width $3\Delta x$, i.e., total 12000 ($8\times500\times3$) sites, while we apply a signal to only 36 sites of the system in Fig. 5. Another interesting work is done by Aranson, Levine, and Tsimring [9], who realized suppressing spatiotemporal chaos [the same model as used in this paper, Eqs. (1) with $\epsilon=0.08$] by developing a spiral wave with local feedback (close loop control) injection. However, there is a key difference between spiral waves and target waves: target waves are not topological defects while spiral waves are topological defects (which sometime are harmful, for example, in cardiac muscle). Another significant difference is the final state of the system after we turn off the signals starting from well-controlled spiral waves and target waves. As it is known, after we turn off the control signal used in Ref. [9], spiral waves (the parameters of the system is in the turbulence region) will not be stable and will break up. And the system will come back to spatiotemporal chaos. On the contrary, target waves will not break up and the whole system will evolve to the spatially homogeneous steady state after we turn off the external force.

In conclusion, we have developed a different and practical method for suppressing spiral waves and spatiotemporal chaos in excitable media. Our method has the attractive feature that a periodic force is applied to the two-dimensional spatiotemporal system only at a single point. By generating target waves with local injection, we have realized controlling spiral waves and spatiotemporal chaos to target waves state or to the spatially homogeneous steady state by releasing the injection after the target waves are achieved. The high efficiency of our control method is due to the fact that the locally stimulated target waves can transmit the control effect from the injected region to the injection-free regions during their outward propagation. Our control method is not model dependent and it works for the model for catalytic CO oxidation on Pt(110) (1) as well as for the model for cardiac tissue (2). We have found that our control scheme also works for the oscillatory medium [when $b<-0.01$ in Eqs. (1)] and the complex Ginzburg-Landau equation which can be derived universally in the vicinity of a homogeneous Hopf bifurcation in extended systems [17]. We hope our work will be of interest in some important practical applications, such as controlling chemical turbulence in catalytic CO oxidation on Pt(110) [12] and ventricular fibrillation in cardiac muscle [11].

ACKNOWLEDGMENTS

This work was supported in part by the grants from the Hong Kong Research Grants Council (RGC), the Hong Kong Baptist University Faculty Research Grant (FRG), the National Nature Science Foundation of China, and the Nonlinear Science Project of China. We would like to thank Professor Qi Ouyang, Dr. Hongliu Yang, and Dr. Jinhua Xiao for useful discussions.

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